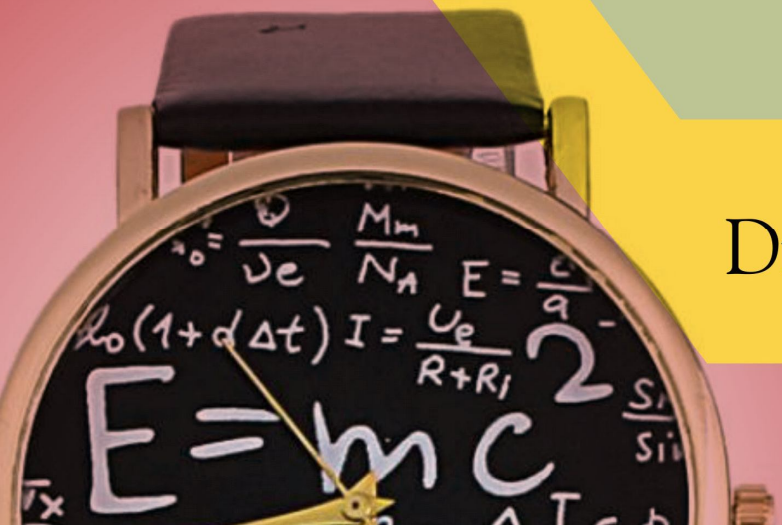
As per the Revised Syllabus of BMS, 2016-17, Semester II,
University of Mumbai

BUSINESS MATHEMATICS



Dr. Abhilasha S. Magar

BUSINESS MATHEMATICS

(As per the Revised Syllabus of BMS, 2016-17, Semester II, University of Mumbai)

Dr. (Mrs.) Abhilasha S. Magar
M.Sc. (MATHS), B.Ed., Dip. C.P., Ph.D.
Department of Mathematics & Statistics,
Annasaheb Vartak College of Arts,
K.M. College of Commerce &
E.S. Andrades College of Science,
Vasai Road (W), Thane – 401202.

Himalaya Publishing House
ISO 9001:2008 CERTIFIED

© **Author**
No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording and/or otherwise without the prior written permission of the publisher.

First Edition : 2017

Published by	:	Mrs. Meena Pandey for **Himalaya Publishing House Pvt. Ltd.,** "Ramdoot", Dr. Bhalerao Marg, Girgaon, Mumbai - 400 004. **Phone:** 022-23860170/23863863; **Fax:** 022-23877178 **E-mail:** himpub@vsnl.com; **Website:** www.himpub.com
Branch Offices	:	
New Delhi	:	"Pooja Apartments", 4-B, Murari Lal Street, Ansari Road, Darya Ganj, New Delhi - 110 002. Phone: 011-23270392, 23278631; Fax: 011-23256286
Nagpur	:	Kundanlal Chandak Industrial Estate, Ghat Road, Nagpur - 440 018. Phone: 0712-2738731, 3296733; Telefax: 0712-2721216
Bengaluru	:	Plot No. 91-33, 2nd Main Road Seshadripuram, Behind Nataraja Theatre, Bengaluru - 560020. Phone: 080-41138821; Mobile: 9379847017, 9379847005
Hyderabad	:	No. 3-4-184, Lingampally, Beside Raghavendra Swamy Matham, Kachiguda, Hyderabad - 500 027. Phone: 040-27560041, 27550139
Chennai	:	New-20, Old-59, Thirumalai Pillai Road, T. Nagar, Chennai - 600 017. Mobile: 9380460419
Pune	:	First Floor, "Laksha" Apartments, No. 527, Mehunpura, Shaniwarpeth (Near Prabhat Theatre), Pune - 411 030. Phone: 020-24496323, 24496333; Mobile: 09370579333
Lucknow	:	House No. 731, Shekhupura Colony, Near B.D. Convent School, Aliganj, Lucknow - 226 022. Phone: 0522-4012353; Mobile: 09307501549
Ahmedabad	:	114, "SHAIL", 1st Floor, Opp. Madhu Sudan House, C.G. Road, Navrang Pura, Ahmedabad - 380 009. Phone: 079-26560126; Mobile: 09377088847
Ernakulam	:	39/176 (New No. 60/251), 1st Floor, Karikkamuri Road, Ernakulam, Kochi - 682011. Phone: 0484-2378012, 2378016; Mobile: 09387122121
Bhubaneswar	:	5, Station Square, Bhubaneswar - 751 001 (Odisha). Phone: 0674-2532129; Mobile: 09338746007
Kolkata	:	108/4, Beliaghata Main Road, Near ID Hospital, Opp. SBI Bank, Kolkata - 700 010. Phone: 033-32449649; Mobile: 7439040301
DTP by	:	**Hansa Bhoir**
Printed at	:	M/s. Seven Hills Printers, Hyderabad. On behalf of HPH.

Preface

I am very happy to present to ***"Business Mathematics"*** (F.Y. BMS Sem-II) to the students of F.Y. BMS (Sem-II). The book is based on the new syllabus applicable from the current academic year 2016-2017. The text closely follows the Chapters, Topic patterns prescribed in the syllabus by Mumbai University.

I have tried to cover the topics in sufficient depth and in simple manner with easy to understand examples.

Special care taken to explain every new concept in minute detail so even those students, who have not studied mathematics at the H.S.C. level will also find it easy to understand the topics covered in the book.

I have tried to present the subject in a simple manner, making use of illustrative examples.

I would like to thank our principal **Dr. Keshav N. Ghorude** for providing an environment which stimulates new thinking and innovation. I would also like to thank self-finance Incharge **Dr. Aravid Ubale** and P.G. Courses Incharge **Dr. G.C. Savagao** for their support.

I would like to thanks my senior and my colleagues for the encouragement and continued support for completing this book.

Special thanks to my husband **Dr. Shashikant Magar** and my loving sons Samarth and Shaurya.

Nothing is perfect in this world and there are always chances to improve. I shall always remain obliged for any improvement suggestions.

A big thanks to **Mr. Srivastav** and all staff of **Himalaya Publishing House Pvt. Ltd.** for bringing out this book in time. Special thanks to **Mr. Santosh Prabhu** and **Mrs. Archana Gupte.**

Dr. (Mrs.) Abhilasha S. Magar

Syllabus

Modules at a Glance

Sr. No.	Modules	No. of Lectures
1	Elementary Financial Mathematics	15
2	Matrices and Determinants	15
3	Derivatives and Applications of Derivatives	15
4	Numerical Analysis (Interpolation)	15
	Total	**60**

1. **Elementary Financial Mathematics**
   - **Simple and Compound Interest:** Interest compounded once a year, more than once a year, continuous, nominal and effective rate of interest.
   - Annuity – Present and future value – sinking funds.
   - **Depreciation of Assets:** Equated Monthly Installments (EMI) using flat interest rate and reducing balance method.
   - **Functions:** Algebraic functions and the functions used in business and economics, Break-even and Equilibrium point.
   - **Permutation and Combination:** Simple problems to be solved with the calculator only.
2. **Matrices and Determinants**
   - **Matrices:** Some important definitions and some important results. Matrix operation (addition, scalar multiplication, matrix multiplication, transpose of a matrix).
   - **Determinants of a matrix of order two or three:** Properties and results of determinants.
   - Solving a system of linear equations using Cramer's rule.
   - Inverse of a Matrix (up to order three) using adjoint of a matrix and matrix, inversion method.
   - **Case Study:** Input-Output Analysis.
3. **Derivatives and Applications of Derivatives**
   - **Introduction and Concept:** Derivatives of constant function, logarithmic functions, polynomial and exponential function.
   - **Rules of derivatives:** Addition, multiplication, quotient.
   - Second order derivatives.
   - **Application of Derivatives:** Maxima, Minima, Average Cost and Marginal Cost. Total revenue, Marginal revenue, Average revenue. Average and Marginal profit. Price elasticity of demand.
4. **Numerical Analysis (Interpolation)**
   - **Introduction and Concept:** Finite differences – forward difference operator – Newton's forward difference formula with simple examples.
   - **Back Difference Operator:** Newton's backward interpolation formula with simple examples.

Paper Pattern

Duration: 2½ Hours **Maximum Marks:** 75

Questions to be Set: 05

All questions are compulsory carrying 15 Marks each.

Question No.	Particulars	Marks
Q.1.	Objective Questions (a) Sub-questions to be asked (10) and to be answered (any 08) (b) Sub-questions to be asked (10) and to be answered (any 07) (*Multiple Choice/True or False/Match the Columns/Fill in the Blanks)	**(15 Marks)**
Q.2.	Full Length Practical Question ***OR*** Full Length Practical Question	**(15 Marks)** **(15 Marks)**
Q.3.	Full Length Practical Question ***OR*** Full Length Practical Question	**(15 Marks)** **(15 Marks)**
Q.4.	Full Length Practical Question ***OR*** Full Length Practical Question	**(15 Marks)** **(15 Marks)**
Q.5. Q.5	(a) Theory Question (b) Theory Question ***OR*** Short Notes To be asked (05) To be answered (03)	**(08 Marks)** **(07 Marks)** **(15 Marks)**

Note: Practical question of 15 Marks may be divided into two sub questions of 7/8 and 10/5 Marks. If the topic demands, instead of practical questions, appropriate theory question may be asked.

Contents

1.	Simple and Compound Interest	1 – 17
2.	Annuity	18 – 31
3.	Depreciation of Assets	32 – 36
4.	Functions	37 – 58
5.	Permutations and Combinations	59 – 72
6.	Matrices and Determinants	73 – 102
7.	Input-Output Analysis	103 – 110
8.	Derivatives and Application of Derivatives	111 – 145
9.	Numerical Analysis	146 – 160

Chapter 1

Simple and Compound Interest

INTRODUCTION

The charge for the privilege of borrowing money is called Interest. In Economics, interest is the return to capital achieved over time.

In banking sector, interest is the extra money that a bank gives you for saving or depositing your money with them.

Similarly, when anybody borrow money, they pay interest.

Mainly there are two types of interest:

1. Simple Interest
2. Compound Interest

Simple Interest: Simple interest is interest calculated only on the initial amount that you invested. The interest charge is always based on the original principal.

To calculate simple interest we use formula

$$\text{S. I.} = \frac{p \times n \times r}{100}$$

Where, p = Principal

n = Time (period)

r = Rate of Interest

If we calculate amount after some period we use form.

$A = SI + P$ or $A = P + SI$ or

$$\therefore A = P + \frac{p \times n \times r}{100} \text{ or}$$

$$A = P\left(1 + \frac{n.r}{100}\right)$$

Note: Banks usually charge compound interest not simple interest

Some Solved Examples

Example 1: Calculate Simple interest on ₹ 10,000 for 3 years at 5% rate of interest per annum.

Solution: Given that

p = 10,000, n = 3 years, r = 5%

We know that S. I. = $\frac{p \times n \times r}{100}$

$\therefore$ S. I. = $\frac{10,000 \times 3 \times 5}{100} = 1500$

$\therefore$ Simple Interest is ₹ 1500.

Example 2: At what rate of simple interest will ₹ 8,000 become ₹ 9,250 in 3 years?

Solution: Given P = 8,000, A = 9,250, n = 3 years.

We know that

$$A = p\left(1 + \frac{n.r}{100}\right)$$

$$9,250 = 8,000\left(1 + \frac{3r}{100}\right)$$

$$\frac{9,250}{8,000} = 1 + \frac{3r}{100}$$

$$1.15625 - 1 = \frac{3r}{100}$$

$$\therefore \frac{3r}{100} = 0.15625$$

$$3r = 15.625$$

$$\therefore \quad r = \frac{15.625}{3}$$

$$\therefore \quad r = 5.2083\%.$$

Example 3: In how many years will a sum of ₹ 12,300 give ₹ 15,000 at 8.5% rate of simple interest?

Solution: Given P = 12,300, A = 15,000, r = 8.5%

We know $A = p\left(1 + \frac{nr}{100}\right)$

$$15,000 = 12,300\left(1 + \frac{8.5.n}{100}\right)$$

$$\frac{15,000}{12,300} = 1 + \frac{8.5n}{100}$$

$$1.2195 - 1 = \frac{8.5n}{100}$$

$0.2195 \times 100 = 8.5n$

$$\therefore n = \frac{21.95}{8.5} = 2.58 \text{ years.}$$

Example 4: In how many years will sum of money be tripled at 20% per annum simple interest?

Solution: Given A = 3P, r = 20%

We know that

$$A = P\left(1 + \frac{nr}{100}\right)$$

$$3p = P\left(1 + \frac{20n}{100}\right)$$

$3 = 1 + 0.2n$

$0.2n = 2$

$$\therefore n = \frac{2}{02} = 10 \text{ years.}$$

Example 5: A sum of ₹ 25,000 is invested at 7% p.a. Find the amount after 5 years.

Solution: Given: P = 25,000, r = 7%, n = 5 years

We know that

$$A = P\left(1 + \frac{nr}{100}\right)$$

$$A = 25{,}000\left(1 + \frac{7 \times 5}{100}\right)$$

A = 25,000 (1.35) = 33,750.

Example 6: Find the present value at simple interest rate of 10% p.a. of ₹ 20,000 payable at the end of 5 years.

Solution: Given

A = 20,000, r = 10%, n = 5 years

We know that

$$A = p\ 1 + \frac{nr}{100}$$

$$20{,}000 = p\left(1 + \frac{10 \times 5}{100}\right)$$

20,000 = p(1.5)

$\therefore p = \frac{20,000}{1.5} = 13,333.3333$

$\therefore$ Present value (p) = ₹ 13,333.3333.

Example 7: Find the simple interest on ₹ 5000 from 3rd Jan. 2015 to 21st Oct. 2015 at 5% rate of interest per annum.

Solution: Given P = 5,000, r = 5%, n = 3rd Jan to 21st Oct 2015

Month	**No. of day**
January	28
February	28
March	31
April	30
May	31
June	30
July	31
August	31
September	30
October	21

$\therefore$ Total no. of days = 291

$\therefore n = \left(\frac{291}{365}\right)$ years

$\therefore SI = \frac{p \times n \times r}{100} = \frac{5,000 \times \left(\frac{291}{365}\right) \times 5}{100}$

$\therefore$ SI = ₹ 199.3150.

Example 8: Nilu took a loan of ₹ 1,200 with simple interest for as many years as the rate of interest. If she paid ₹ 432 as interest at the end of the loan period what was the rate of interest?

Solution: Given p = 1,200, SI = 432, r = n

We know that

$$SI = \frac{p \times n \times r}{100}$$

$$432 = \frac{1,200 \times r \times r}{100}$$

$$43,200 = 1,200 \times r^2$$

$$\frac{43,200}{1,200} = r^2$$

$$r^2 = 36$$

$\therefore$ r = 6%.

Example 9: A sum of money amounts to ₹ 25,000 after 3 years and ₹ 25,000 after 5 years at the same rate of simple interest. Find the rate of simple interest per annum.

Solution: S.I. for 2 years = 25,000 – 20,500 = ₹ 4,500

Now S. I. for 3 year $\left(\frac{4,500}{2}\times 3\right)$

= ₹ 6,750

∴ Principal = 20,500 – 6,750 = ₹ 13,750

We know that

$$SI = \frac{p \times n \times r}{100}$$

$$6,750 = \frac{13,750 \times 3 \times r}{100}$$

∴ 67,5,000 = 41,250r

$$\therefore \quad r = \frac{6,75,000}{41,250} = 16.36\%$$

∴ = 16.36%.

Example 10: Milind wants to have an interest income of ₹ 5,000 a year. How much he invest for one year at 7.5%?

Solution: Given SI = ₹ 5,000, n = 1 year, r = 7.5%

$$SI = \frac{p \times n \times r}{100}$$

$$5,000 = \frac{p \times 1 \times 7.5}{100}$$

$$\therefore \quad p = \frac{5,000 \times 100}{1 \times 7.5}$$

∴ p = ₹ 66,666.6666

∴ He most invest ₹ 66,666.6666.

Example 11: Joel lent ₹ 10,000 for 3 years and ₹ 7,500 for 2 years, at the same rate of simple interest. If he received ₹ 4,500 as interest, find the rate of simple interest.

Solution: Given

I case p = ₹ 10, 000, n = 3 years

II case p = 7,500, n = 2 years

SI = ₹ 4,500

We know that $SI = \frac{p \times n \times r}{100}$

For case I

$$SI = \frac{10,000 \times 3 \times r}{100}$$

$$SI = 300\ r \qquad ...(1)$$

For case II

$$SI = \frac{7,500 \times 2 \times r}{100} = 150r \qquad ...(2)$$

From (1) and (2)

$$SI = 300\ r + 150\ r$$

$$4,500 = 450\ r$$

$$\therefore \quad r = \frac{4,500}{450} = 10\%$$

$$\therefore \quad r = 10\%.$$

Example 12: Determine the simple interest rate applied to a principal over 25 years if the total interest paid equals the borrowed principal.

Solution: Given that SI = P, n = 25 years

We know that $SI = \frac{p \times n \times r}{100}$

$$P = \frac{p \times 25 \times r}{100}$$

$$\therefore r = \frac{100 \times p}{25 \times p}$$

$$\therefore r = 4\ \%.$$

Example 13: Mr. Navin deposit ₹ 15,000 in one simple interest account and ₹ 12,000 in another simple interest account. The interest rate on first account is 2% more than the second account. If the total simple interest on both account is ₹ 950, find the rate of interest on both account.

Solution:

For I – Account

P = 15,000, n = 1 year r = 2% more than second account

$SI = \frac{p \times n \times r}{100}$ i.e., $r = \left(\frac{2}{100} + r\right) = 0.02 + r$

$$= \frac{15,000 \times 1 \times (0.2 + r)}{100} \text{ by given condition}$$

$$= 150(0.2 + r)$$

$$SI = 30 + 150r. \qquad ...(1)$$

For II – Account

P = 12,000 n = 1

$$\therefore SI = \frac{p \times n \times r}{100}$$

$$= \frac{12{,}000 \times 1 \times r}{100} = 120r \qquad ...(2)$$

From (1) and (2)

SI = (30 + 150r) + (120r)

950 = 30 + 270r

950 – 30 = 270r

920 = 270r

$$\therefore r = \frac{920}{270} = 3.407\%.$$

$\therefore$ The interest rate of I account r = 3.407% and the interest rate of II account r = 5.407%.

EXERCISE

1. Find the amount of simple interest that is paid over a period of 3 years on a principal of ₹ 22,000 at S.I. Rate of 6% p. a.
   (**Ans.:** S.I. = 3,960)
2. Calculate the total amount of an investment of ₹ 12,500 after six month at S.I. rate of 5% p.a.
   (**Ans.:** S.I. = 312.5, A = 12,812.5)
3. At what rate of simple interest will ₹ 9,250 becomes ₹ 11,000 in 2 years?
   (**Ans.:** 9.459%)
4. In how many years a principal payment to double at a S. I. rate of 10% p.a.?
   (**Ans.:** n = 20 years)
5. In how many years will a sum of ₹ 18,000 gives ₹ 19,000 at 5.555% rate of S.I.?
   (**Ans.:** 1 year)
6. Saket took a loan of ₹ 50,000 with simple Interest for as many years as the rate of interest. It he paid ₹ 2,000 as interest at the end of the loan period, what was the rate of interest?
   (**Ans.:** r = 2%)
7. Find the present value at S.I. Rate of 8% per annum of ₹ 35,000 payable at the end of 4 years.
   (**Ans.:** p = ₹ 26,515.1515)
8. Find the S.I. on ₹ 6,000 from 13th Feb 2005 to 29th June 2005 at 6% rate of interest p.a.
   (**Ans.:** S.I. = ₹ 134.136)
9. Mr. PK deposit ₹ 8,000 in first simple interest account and ₹ 2,000 in second simple interest account. The interest rate on first account is 2% more than the second account. If the total simple interest on both account is ₹ 950. Find the rate of interest on both account.
   (**Ans.:** II account rate = 4.18%
   I account rate = 6.18%)

10. Yogesh lent ₹ 20,000 for 5 years, and ₹ 16,000 for 3 years at the same rate of S.I. If he received ₹ 7,400 as interest. Find the rate of S.I.

(**Ans.:** r = 5%)

COMPOUND INTEREST

At the end of every year interest calculated and added to the principal and also on the accumulated interest of previous periods of deposit or loan is called as compound interest. In another words "Interest which is calculated not only on the initial principal but also the accumulated interest of previous period.

Compound interest may be different with simple interest where interest is not added to the principal compound interest is standard in finance. Banks usually charge compounding interest

Formula for calculating compound Interest is given by

CI = A – P

Where, A = Amount (value) after n times

P = Principal (Initial) amount/Present value

Here we calculate A (Accumulate value) is given by

$A = P\left(1 + \frac{r}{100}\right)^{n}$ for yearly/Annually

$A = P\left(1 + \frac{r}{200}\right)^{2n}$ for Semi-yearly/Half-yearly

$A = P\left(1 + \frac{r}{400}\right)^{4n}$ for quarterly

$A = P\left(1 + \frac{r}{1200}\right)^{12n}$ for monthly

Where, r = rate of C.I.

n = time (period)

P = Principal (Initial value)/Present value

A = Amount (Accumulate Value)

$CI = A - P$

$CI = P\left(1 + \frac{r}{100}\right)^{n} - P$

$\therefore CI = P\left[\left(1 + \frac{r}{100}\right)^{n} - 1\right]$

Some Solved Examples

Example 1: A sum of ₹ 10,000 is invested at 10% per annum Compound interest. Find amount after 3 years. Also calculate compound interest.

Solution: Given

P = 10,000, r = 10%, n = 3 years

To calculate A = ?, CI = ?

We know that

$$A = p\left(1 + \frac{r}{100}\right)^n$$

$$A = 10{,}000\left(1 + \frac{10}{100}\right)^3$$

$\therefore$ A = ₹ 13,310

Now CI = A – P

= 13,310 – 10,000

$\therefore$ CI = ₹ 3,310.

Example 2: A bank promises to give you ₹ 20,000 after 4 years at 8% rate of compound interest. Find today's investment.

Solution.: Given A = ₹ 20,000, n = 4 years, r = 8%

We know that

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$20{,}000 = P\left(1 + \frac{8}{100}\right)^4$$

20,000 = P (1.36048)

$$\therefore \quad P = \frac{20{,}000}{1.36048}$$

$\therefore$ P = ₹ 14,700.6938

i.e., Today investment (P) = ₹ 14,700.6938.

Example 3: The simple interest and compound interest on a sum of money at a certain rate for 2 years is ₹ 2,200 and 2,850 respectively. Find the rate and the sum.

Solution: Given SI = 2,200, CI = 2,850, n = 2 years

We know that $SI = \frac{p \times n \times r}{100}$

$$2{,}200 = \frac{p \times 2 \times r}{100}$$

$\therefore$ 22,000 = 2 p.r.

$$Pr = \frac{22{,}000}{2}$$

$\therefore$ Pr = 1,10,000 ...(1)

$$\text{Also CI} = P\left[\left(1 + \frac{r}{100}\right)^n - 1\right]$$

$$2{,}850 = P\left[\left(1 + \frac{r}{100}\right)^2 - 1\right] \quad ...(2)$$

Dividing (2) by (1)

$$\frac{2{,}850}{1{,}10{,}000} = \frac{p\left[\left(1 + \frac{r}{100}\right)^n - 1\right]}{P.r.}$$

$$0.02590 = \frac{\left[1 + \frac{2r}{100} + \frac{r^2}{10{,}000} - 1\right]}{r}$$

$$0.02590 = \frac{\left[\frac{200r + r^2}{10{,}000}\right]}{r}$$

$$0.\ 02590 = \frac{r(200 + r)}{10{,}000r}$$

$$0.02590 = \frac{200 + r}{10{,}000}$$

$\therefore$ 200 + r = 259

$\therefore$ r = 59% Put in (1)

$\therefore$ P (59) = 1,10,000

$$\therefore P = \frac{1{,}10{,}000}{59} = 1{,}864.406$$

$\therefore$ Rate (r) = 59% and sum (P) = ₹ 1,864.406.

Example 4: Find the accumulated value at the end of 2 years of ₹ 8,000 invested at 7% p.a. compounded half yearly.

Solution: Given, n = 2 years, P = 8,000, r = 7% p.a.

We know that,

$$A = P\left(1 + \frac{r}{200}\right)^{2n}$$

$$A = 8{,}000\left(1 + \frac{7}{200}\right)^{2(2)}$$

$= 8{,}000\ (1.147523)$

A = ₹ 9,180.184

i.e., Accumulate value after 2 years

A = ₹ 9,180.184.

Example 5: The difference between simple interest and compound interest on a certain sum for 3 years at 6% p.a. is ₹ 100. Find the sum.

Solution: Given: n = 3 years, r = 6%, CI – SI = 100

We know that CI > S.I.

$$\text{Also SI} = \frac{P \times n \times r}{100}$$

$$= \frac{P \times 3 \times 6}{100}$$

$\therefore \quad SI = 0.18P$ ...(1)

$$CI = P\left[\left(1 + \frac{r}{100}\right)^{n} - 1\right]$$

$$= P\left[\left(1 + \frac{6}{100}\right)^{3} - 1\right]$$

CI = 0.1910 P ...(2)

By given condition CI – SI = 100

(0.1910P) – (0.18P) = 100

$$0.011\ P = 100 \quad \therefore p = \frac{100}{0.011}$$

$\therefore$ p = ₹ 9,090.9090.

Example 6: At what rate per cent per annum a sum of ₹ 1,225 to ₹ 1,600 after 2 year compounded annually?

Solution: Given

P = 1,225, A = 1,600, n = 2 years

We know that

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$1,600 = 1,225\left(1 + \frac{r}{100}\right)^2$$

$$\frac{1,600}{1,225} = \left(1 + \frac{r}{100}\right)^2$$

$$\left(\frac{40}{35}\right)^2 = \left(1 + \frac{r}{100}\right)^2$$

Since $\left(\frac{40}{35}\right)^2 = \frac{1,600}{1,225}$

$$\Rightarrow \quad \frac{40}{35} = 1 + \frac{r}{100}$$

$$1.1428 - 1 = \frac{r}{100}$$

$\therefore r = 0.1428 \times 100$

$\therefore r = 14.285\%$.

Example 7: A sum of ₹ 1,000 to ₹ 1,728 in 3 years compound interest. Find the rate.

Solution: Given P = 1,000, A = 1,728, n = 3 years

We know that

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$1,728 = 1,000\left(1 + \frac{r}{100}\right)^3$$

$$\frac{1,728}{1,000} = \left(1 + \frac{r}{100}\right)^3$$

$$\left(\frac{12}{10}\right)^3 = \left(1 + \frac{r}{100}\right)^3 \qquad \therefore \left(\frac{12}{10}\right)^3 = \frac{1,728}{1,000}$$

$$\therefore \frac{12}{10} = 1 + \frac{r}{100}$$

$$1.2 - 1 = \frac{r}{100}$$

$\therefore \quad r = 0.2 \times 100 \qquad \therefore r = 20\%$.

Example 8: The difference between S.I. & CI. on an account of ₹ 32,000 for 2 years is ₹ 150. Find the rate per cent per annum.

Solution.: Given p = 32,000, n = 2 years, CI – SI = 150

We know that

$$SI = \frac{P \times n \times r}{100}$$

$$= \frac{32{,}000 \times 2 \times r}{100}$$

$$\therefore SI = 640r \qquad ...(1)$$

$$\text{Also } CI = P\left[\left(1 + \frac{r}{100}\right)^n - 1\right]$$

$$= 32{,}000\left[\left(1 + \frac{r}{100}\right)^2 - 1\right]$$

$$= 32{,}000\left[1 + \frac{2r}{100} + \frac{r^2}{10{,}000} - 1\right]$$

$$= 32{,}000\left[\frac{200r + r^2}{10{,}000}\right]$$

$$= 3.2\,[200r + r^2]$$

$$CI = 640r + 3.2r^2 \qquad ...(2)$$

By given condition

CI – SI = 150 $\quad \therefore$ From (1) and (2)

$(640r + 3.2r^2) – (640r) = 150$

$\therefore\ 3.2\ r^2 = 150$

$$r^2 = \frac{150}{3.2}$$

$r^2 = 46.875$

$\therefore r = 6.846\%$.

Example 9: In how many years would ₹ 23,500 become ₹ 29,750 at 6% p.a. compound interest?

Solution.: Given P = 23,500, A = 29,750, r = 6%

We know that

$$A = p\left(1 + \frac{r}{100}\right)^n$$

$$29,750 = 23,500\left(1 + \frac{6}{100}\right)^n$$

$$\frac{29,750}{23,500} = (1.06)^n$$

$$(1.26) = (1.06)^n$$

$$(1.06)^4 = (1.06)^n$$

$\therefore$ n = 4 years.

Example 10: The compound interest on ₹ 30,000 at 7% p.a. is ₹ 4,347. Find period in years.

Solution: Given P = 30,000, r = 7%, CI = 4,347

We know that CI = A – P

$\therefore$ A = CI + P

A = 4,347 + 30,000

$\therefore$ A = 34,347.

$$\text{Now } A = P\left(1 + \frac{r}{100}\right)^n$$

$$34,347 = 30,000\left(1 + \frac{7}{100}\right)^n$$

$$\frac{34,347}{30,000} = (1.07)^n$$

$$(1.1449) = (1.07)^n$$

$$(1.07)^2 = (1.07)^n$$

$\therefore$ n = 2 years.

Example 11: If you deposit ₹ 5,000 in an account paying 8% annual interest compounded quarterly for 2 years. Find the amount after 2 years.

Solution: Given p = ₹ 5,000, r = 8% p.a. comp. quarterly

n = 2 years.

we know that, for compounded quarterly

$$A = P\left(1 + \frac{r}{100}\right)^{4n}$$

$$= 5,000\left(1 + \frac{8}{400}\right)^{4(2)}$$

$$= 5,000\ (1.1716)$$

A = 5,858

$\therefore$ Amount after 2 years A = ₹ 5,858.

Example 12: The simple interest on a sum of money for 4 years at 5.5% p.a. is ₹ 8,570. what will be the compound interest on the same at the same rate for the same period compound annually?

Solution: Given n = 4 years, r = 5.5%, SI = ₹ 8,570

We know that,

$$SI = \frac{P \times n \times r}{100}$$

$$8,570 = \frac{p \times 4 \times 5.5}{100}$$

$$\therefore P = \frac{8,570 \times 100}{4 \times 5.5}$$

$$\therefore P = ₹\ 38,954.5454$$

$$\text{Now CI} = P\left[\left(1 + \frac{r}{100}\right)^n - 1\right]$$

$$= 38,954.5454\left[\left(1 + \frac{5.5}{100}\right)^4 - 1\right]$$

$$= 9,303.3056$$

$$\therefore \quad CI = ₹\ 9,303.3056.$$

Example 13: Find the difference between S.I. and C.I. on ₹ 5,050 for two year at 10% p.a.

Solution: We know that $SI = \frac{P \times n \times r}{100}$

$$= \frac{5,050 \times 2 \times 10}{100}$$

$$SI = 1,010 \qquad ...(1)$$

Also

$$CI = P\left[\left(1 + \frac{r}{100}\right)^2 - 1\right]$$

$$= 5,050\left[\left(1 + \frac{10}{100}\right)^2 - 1\right]$$

$$= 5,050\ [0.21]$$

$$CI = 1,060.5 \qquad ...(2)$$

From (1) and (2)

$$CI - SI = 1,060.5 - 1010$$

$$\therefore CI - SI = ₹\ 50.5.$$

Example 14: The difference between S.I. and C.I. on a ceratin sum of money for 2 years at 5% p.a. is ₹ 1 find the sum.

Solution: Given n = 2 year, r = 5% CI – SI = ₹ 1

We know that SI = $\frac{P \times n \times r}{100} = \frac{P \times 2 \times 5}{100}$

$\therefore$ SI = 0. I P ...(1)

Also CI = $P\left[\left(1+\frac{r}{100}\right)^{n}-1\right]$

$= P\left[\left(1+\frac{5}{100}\right)^{2}-1\right]$

CI = 0.1025P ...(2)

By given cond. CI – SI = 1

0.1025P – 0.1P = 1

0.0025P = 1

$\therefore P = \frac{1}{0.025} = 400$ $\therefore$ The sum (P) = ₹ 400.

EXERCISE

1. Calculate the amount after 5 years a principal of ₹ 18,000 invested at 7% per annum compound interest.
   (**Ans.:** A = ₹ 25,245.93115)
2. A bank promises to give you ₹ 1,00,000 after 3 years at the rate of 8% per annum compound interest. Find todays investment. Also calculate compound interest
   (**Ans.:** P = ₹ 79,383.22 CI = 20,616.7759)
3. At what rate per cent per annum a sum of ₹ 3,600 to ₹ 2,500 after 2 years compounded annually?
   (**Ans.:** r = 20%)
4. Find rate of interest if a sum of ₹ 1,000 to ₹ 1,360.48 after 4 years compound interest.
   (**Ans.:** r = 8% p.a.)
5. In what time will ₹ 2,000 becomes ₹ 2,662 at 10% p.a. compounded annually?
   (**Ans.:** n = 3 years.)
6. How many time periods in year are needed to increase ₹ 3,000 at 8.5% p.a. compound interest to ₹ 4,500.
   (**Ans.:** n - 5 years (approximately)
7. Find the difference between S.I. and C.I. at 6% p.a. on the sum of ₹ 10,000 after 4 years.
   (**Ans.:** CI – SI = 2,624.7696 – 2,400 = ₹ 2,24.7696)

8. Find the C.I. on ₹ 7,500 at 4% p.a. for 2 years compounded annually.

   (**Ans.:** CI = 612)

9. How much money would you need to deposit today at 8% p.a. compounded monthly to have ₹ 40,000 in the account after 2 years.

   (**Ans.:** P = ₹ 34,104.085)

10. The difference between compound interest and simple interest on an account of ₹ 14,000 for 2 years is ₹ 120. Find the rate per cent.

    (**Ans.:** r = 9.258%)

11. The simple interest and compound interest on a sum of money at a certain rate for 2 years is ₹ 5,500 and 6,600 respectively. Find the rate and sum.

    (**Ans.:** r = 40% P = ₹ 6,875)

OBJECTIVE QUESTIONS

State whether the following Statement is True/False

1. Compound interest is the interest earned not only on the original principal.
2. The sum of principal and the interest is called amount.
3. In financial analysis we often come across uneven cash flow streams.
4. Compound interest for a principal is less than the simple interest on the same amount.
5. The process of discounting used for calculating the present value is simply the inverse of compounding.

Ans.: 1. False, 2. True, 3. True, 4. False, 5. True.