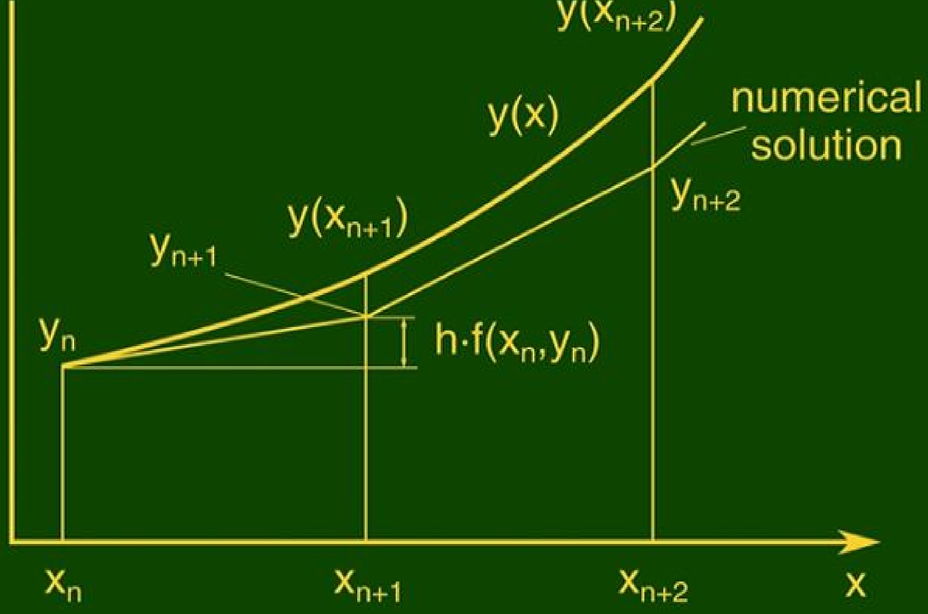
NUMERICAL AND STATISTICAL METHODS

Bhupendra T. Kesaria



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PREFACE

Dear Students,

I am extremely happy to come out with this book on "*Numerical and Statistical Methods*". The topics within the chapters have been arranged in a proper sequence to ensure smooth flow of the subject. Large number of solved examples are included in all chapters for better understanding of students. I sincerely hope that this book will cater to all your needs in this subject.

I thank my daughter Ekta B. Kesaria for her help in preparing and solving the problems. I also thank my family for their encouragement and support.

I also thank Mr. Srivastava of Himalaya Publishing House Pvt. Ltd. and his entire staff for their efforts in publishing this book. We have jointly made every possible effort to eliminate all the errors in the book, however if you find any, please let us know, because that will help us to improve further.

I am grateful to Dr. Mrs. Anju Kapoor (Principal, Usha Pravin Gandhi College of Management) for her constant encouragement, influence and relentless support.

I thank Prof. Hirendand (Coordinator, B.Sc. I.T., Mulund College of Commerce), Prof. Smruti Nanavaty (Coordinator, M.Sc., I.T., Usha Pravin Gandhi College of Management) and Prof. Swapnali Lotlikar (Coordinator, B.Sc. I.T., Usha Pravin Gandhi College of Management) for their inspiration and support.

I also thank my friends and colleagues for their encouragement and patience.

- Author

CONTENTS

1 MATHEMATICAL MODELING AND ENGINEERING PROBLEM SOLVING

STRUCTURE:

1.0. INTRODUCTION

Knowledge and understanding are prerequisites for the effective implementation of any tool. This is particularly true when using computers to solve engineering problems. Although they have great potential utility, computers are practically useless without a fundamental understanding of how engineering system works.

This understanding is initially gained by empirical means that is by observation and experiment. However, while such empirically derived information is essential, it is only half the story. Over years and years of observation and experiments; engineers and scientists have noticed that certain aspects of their empirical studies occur repeatedly.

The primary objective of this chapter is to introduce you to mathematical modeling and its role an engineering problem solving. We will also illustrate how numerical methods figure in the process.

1.1 OBJECTIVES

To provide a concrete idea of what numerical methods are and how they relate to engineering and scientific problem solving.

- Learning how mathematical models can be formulated on the basis of scientific principles to simulate the behaviour of a simple physical system.
- Understanding how numerical methods afford a means to generalize solutions in a manner that can be implemented on a digital computer.
- Understanding the different types of conservation laws that lie beneath the models used in the various engineering disciplines and appreciating the difference between steady-state and dynamic solutions of these models.
- Learning about the different types of numerical methods we will cover in next section.

1.2 A SIMPLE MATHEMATICAL MODEL

- A mathematical model can be broadly defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms.
- Models can be represented by a functional relationship between dependent variables, independent variables, parameters, and forcing functions.

  Dependent Variables = f (independent variables, parameters, forcing functions)
- Dependent variable – a characteristic that usually reflects the behaviour or state of the system.
- Independent variables – dimensions such as time and space, along which the systems behaviour is being determined.
- Parameters – constants reflective of the systems properties or composition.
- Forcing functions – external influences acting upon the system.

  Modeling : Newton's second Low motion.
- Statement: The time rate of change of momentum of a body is equal to the resultant force acting on it.

$$F = ma$$

$$\Rightarrow a = \frac{f}{m} \longleftarrow \text{forcing function}$$

Dependent variable (acceleration) → a; Parameter (mass of object) → m

- No independent variable is involved.
- Eg. it can be used to determine the terminal velocity of a free-falling body near the earth's surface.
- It describes a natural process or system in mathematical terms.
- It represents an idealization and simplification of reality.
- Ignore negligible details of the natural process and focus on its essential manifestations.
- Exclude the effects of "relativity" that are minimal importance when applied to object and forces that interact on or about the earth's surface at velocities and on scales visible to humans.
- It yields reproducible results and can be used for predictive purposes.
- Have generalization capabilities.

  Bungee – jumping
- For a body falling within the vicinity of the earth, the net force is composed of two opposing forces.

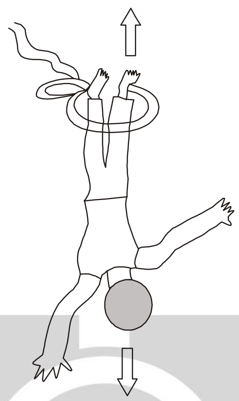
  $F = f_D + f_u$
- The downward Pull of gravity F_D.
  - The force due to gravity can be formulated as:

    $F_D = mg$
  - g is the acceleration due to gravity (9.81 m/s^2)

- The upward force of air resistance f_u
  - A good approximations is to formulate it as:

    $F_u = -CdV^2$
  - V is the velocity; Cd is the lumped drag coefficient, accounting for the properties of the falling object like shape or surface roughness.
  - >>The greater the fall velocity, the greater the upward force due to air resistance

Upward force due to air resistance



Downward force due to gravity

- The net force therefore is the difference between and upward force, we can have a differential equation regarding the velocity of the object.

$$\frac{dv}{dt} = g - \frac{cd}{m}v^2$$

- The exact solution of v cannot be obtained using simple algebraic manipulation but rather using more advanced calculus techniques (when $v(t) = 0$, $t = 0$)

$$v(t) = \sqrt{\frac{gm}{Cd}} \tan h\left[\sqrt{\frac{ged}{m}}t\right]$$

Here t is independent variable, $v(t)$ is dependent variable, Cd and m are parameters.

g is forcing function

$$\tan h\,(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Example 1:

- A bungee jumper with a mass of 68.1 kg leaps from a stationary hot air balloon (the drag coefficient is 0.25 kg/m).
  - Compute the velocity for the first 12s of free fall.

    Determine the terminal velocity that will attained for an infinite long cord.

$$V(t) = \sqrt{\frac{gm}{Cd}} \tan h\left[\sqrt{\frac{gCd}{m}}\ t\right]$$

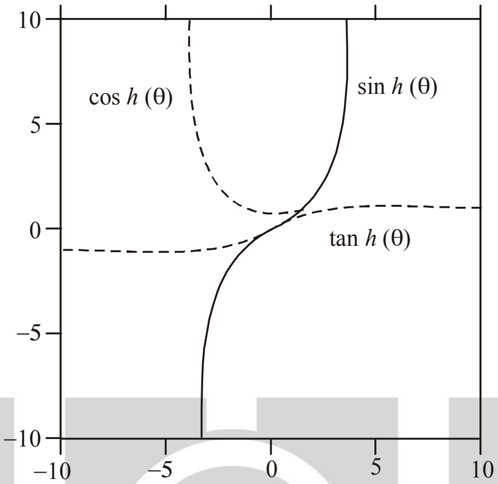
$$V(t) = \sqrt{\frac{9.8\,(68.1)}{0.25}} \tan h \left[\sqrt{\frac{9.8\,(0.25)t}{68.1}} \right] = 51.6938 \tan h\,(0.18977\, t)$$

$$\therefore \quad V(12) = 50.6715$$

$$V(100) \approx 50.6938$$

$mg = cdV^2$

$V = \sqrt{\frac{gm}{Cd}}$

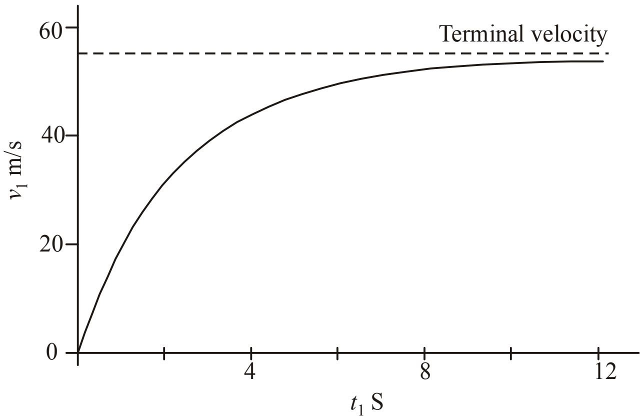


Example 2:

- Using a computer (or a calculator), the model can be used to generate a graphical representation of the system.

$$mg = CdV^2$$

$$V = \sqrt{\frac{gm}{Cd}}$$



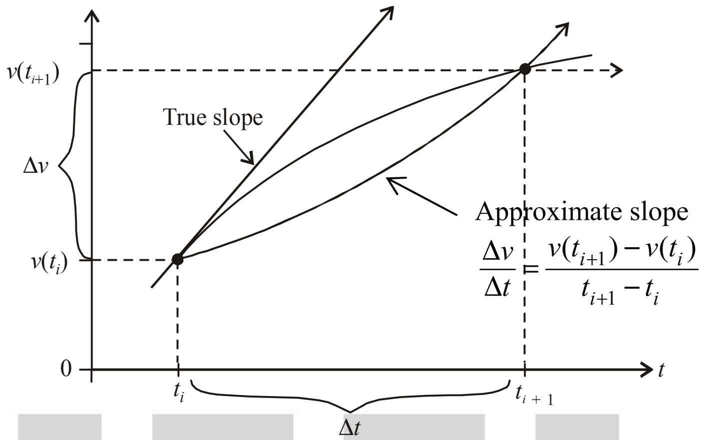
Example of Numerical Modeling

- Numerical methods are those in which the mathematical problem is reformulated so it can be solved by arithmetic operations.

  E.g., the time rate of change of velocity mentioned earlier:

$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$ (a finite-difference approximation of the derivate at time t_i)

Notice that $\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$



. • Substituting the finite difference into the differential equation gives,

$$\frac{dv}{dt} = g - \frac{Cd}{m} v^2$$

$$\Rightarrow \quad \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{cd}{m} v(t_i)^2$$

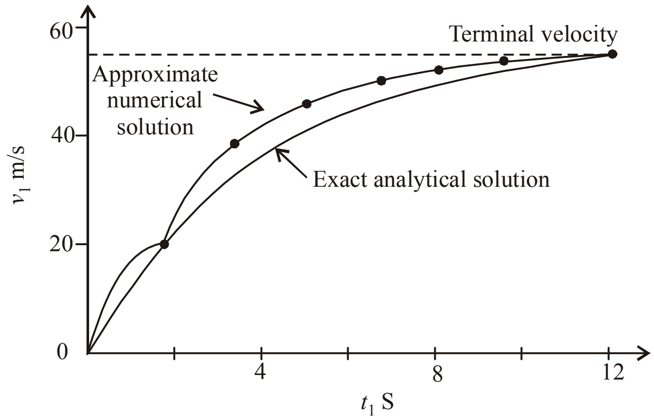
$\Rightarrow$ Solve for

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{Cd \cdot v(t_i)^2}{m} \right] (t_{i+1} - t_i)$$

$$\text{new} = \text{old} + [\textit{Slope} \quad x] \text{ step} \qquad (t_{i+1} - t_i)$$

This approach is formally called Euler's method.

Applying Euler's method in 2*s* intervals yields



How do we improve the solution?

- Smaller steps.

1.3 CONSERVATION LAWS

Conservation laws form the basis of a variety of complicated and powerful model and are conceptually easy to understand.

Conservation laws provide the foundation for many model functions.

— They boil down to

Change = increase – decreases

— Can be used to predict changes with respect to time by given it a special name "the time-variant (or transient)" computation

— If no change occurs, the increases and decreases must be in balance.

Change = 0 = increases – decreases

- It is given a special name, the "steady-state" calculation

Example : Fluid flow

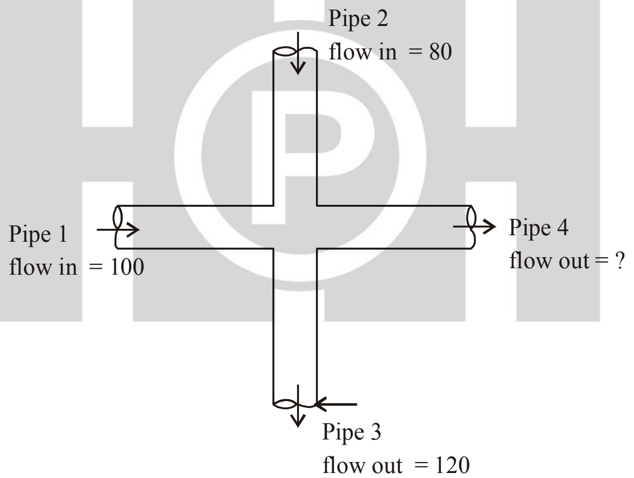


Fig : A flow balance for steady incompressible fluid flow at the junction of pipes.

— For steady-state incompressible fluid flow in pipes flow in = Flow out

- The flow out of the fourth pipe must be 60.

Table 1: Devices and types of balances that are commonly used in the four major areas of engineering.

For each case, the conservation law upon which the balance is based is specified.

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